Application Report
DLPA044–December 2013

Geometric Optics for DLP®

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ABSTRACT

Many frequently asked questions about DLP applications involve **geometric optics**. This application report discusses and answers some of these questions.

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1 Introduction

1.1 Definitions

Geometric optics— The term applied to optical analysis, which considers the propagation of light along ray paths in ordinary 3-dimensional space. It is also called ray optics. It does not consider the wave nature of light, or the quantum nature of light. It is convenient for understanding the spatial characteristics of an optical system, such as projected image size, and image placement. It also addresses factors such as image brightness, focus, and depth of field.

Projector— An optical device which can produce an illuminated image at some distance from itself, usually on a screen of some kind. A screen is not always required. A projector can be used for scene illumination, as in structured light applications.

Screen— The surface where the desired projected image is formed. This is usually a diffusely reflecting surface (Lambertian).

Light engine (LE)— An assembly containing the optical components (lenses and so forth) and illumination devices (commonly LEDs) of a projector. The LE is usually considered separate from the electronics which drive the display element. For DLP systems, the display element is a digital micromirror device (DMD).

Illumination— The source of light which illuminates the projected image. This can be a single or multiple lamps, LEDs, lasers, or hybrid sources. An often used solid state illumination source consists of a set of red, green, and blue LEDs.

2 Aspect Ratio

Aspect ratio is a number which gives the ratio between the width and the height of a DMD. It can be expressed as a ratio, or decimal number. Figure 1 shows three different aspect ratios. All three shapes have the same height, but different widths. The orange square (top left) has an aspect ratio of 1:1. The purple rectangle (top right) has an aspect ratio of 4:3, which is typical of old analog TV screens. The green rectangle (bottom) is 16:9, which is one of the aspect ratios used for HDTV.

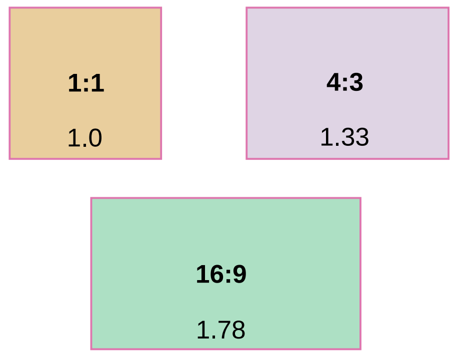

Figure 1. Aspect Ratios

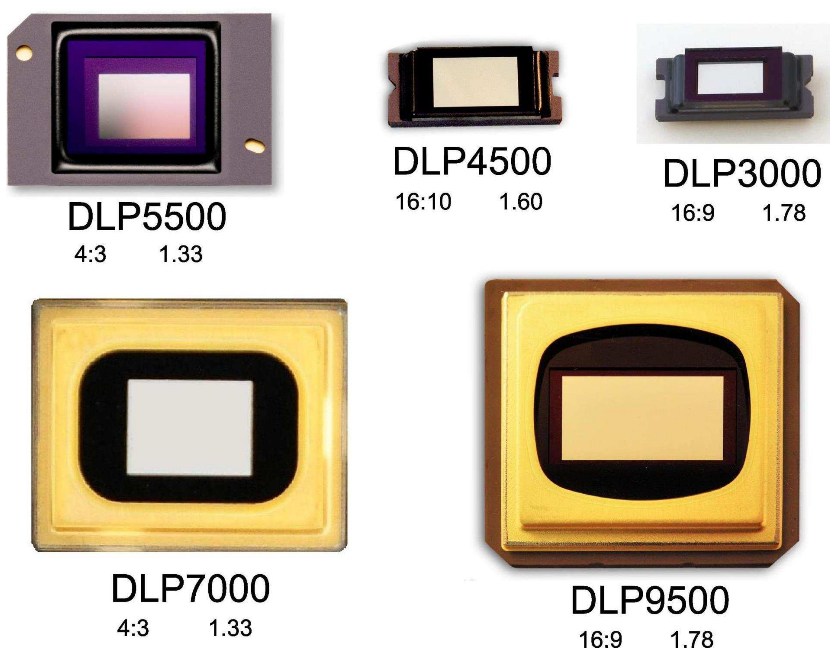

Figure 2. DLP DMDs With Their Aspect Ratios

2.1 *Aspect Ratio of Diamond-Pixel-Array DMD versus Orthogonal-Pixel-Array DMD*

DMDs can have their mirrors arranged parallel or perpendicular to the sides of the chip, or on a diagonal with respect to the sides of the chip. The first arrangement (parallel or perpendicular) is referred to as an orthogonal pixel array DMD. The second arrangement (diagonal) is called a diamond pixel array DMD. For the orthogonal DMDs, the aspect ratio is the ratio of the number of horizontal mirrors to the number of vertical mirrors.

Example: DLP5500 H = 1024, V = 768, 1.33 or 4:3

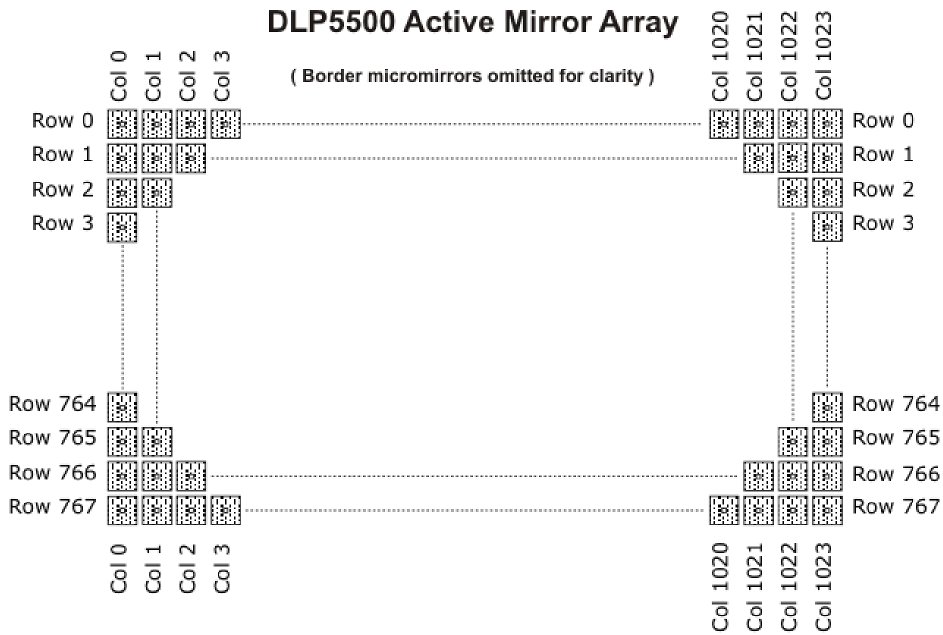

Figure 3. DLP5500 Active Mirror Array

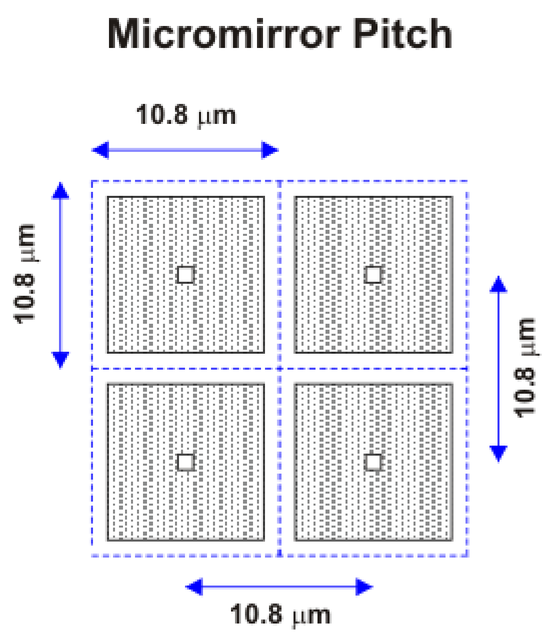

Figure 4. DLP5500 Micromirror Pitch

However, for a diamond pixel array, the aspect ratio is not simply the number of mirrors in the x-dimension divided by the number of mirrors in the y-dimension. The meaning of row and column is different for a diamond pixel array (not because the mirror-to-mirror spacing is different; the entire array is merely rotated by 45°).

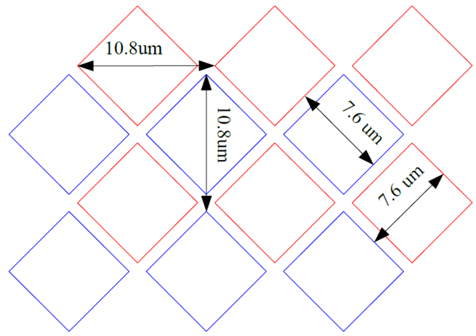

Figure 5. DLP4500 Diamond Pixel Array

For a diamond pixel array, consult the device data sheet on www.ti.com for the linear dimensions of the active area of the DMD. The example of the DLP4500 is given in Figure 6.

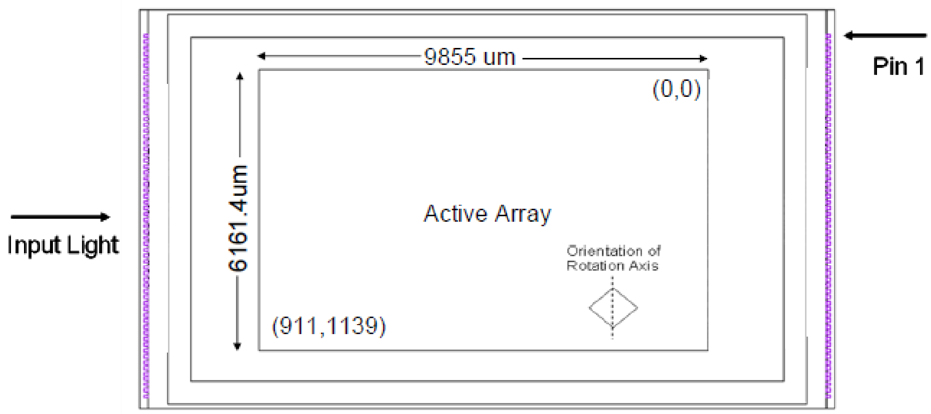

Figure 6. DLP4500 Dimensions of Active Array

2.2 Diagonal Measurement Used for DMDs

DMDs are often referred to by their diagonal measurement in inches.

For example, the diagonal of the DLP4500 is calculated from its active array dimensions by:

$$L = \sqrt{H^2 + V^2} = \sqrt{9855^2 + 6161.4^2} = 11622\ \mu m = 0.4576\ in \quad (1)$$

For this reason, the DLP4500 is called a 0.45” DMD. This is the origin of “4500” in the part number.

3 Complex Lenses versus Simple Lenses

High-quality projection lenses, like photographic lenses, are often complex optical components. To control optical aberrations (spherical, chromatic, astigmatism, coma, and distortion), they are designed with multiple lens elements of different refractive indices and surface curvatures, possibly some with aspheric (non-spherical) surfaces.

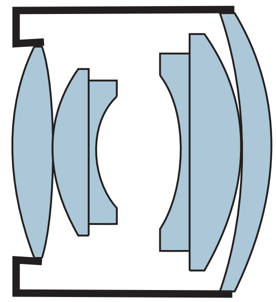

Figure 7. Example of a Complex Photographic Lens

Analyzing a complex lens can be very involved. However, for some visualizations and calculations, a complex lens can be considered in a simplified manner by considering only the location of its entrance pupil and effective aperture.

The entrance pupil is the projection center of the lens, the center of its point-of-view. The effective aperture is the size of the entrance pupil, which determines the amount of light that can pass through a lens, or its "brightness".

A complex lens has both an entrance pupil and an exit pupil, which probably does not correspond in space (that is, both are not in the same location along the optical axis of the lens). However, the concept of an entrance pupil allows the simple calculation of effective focal length and effective aperture for the lens. Therefore, we consider the simplest lens design – a thin lens. For a thin lens, the entrance and exit pupils correspond, and are the same as the stopped aperture of the lens. Although a thin lens is an oversimplification, it suffices for our present considerations.

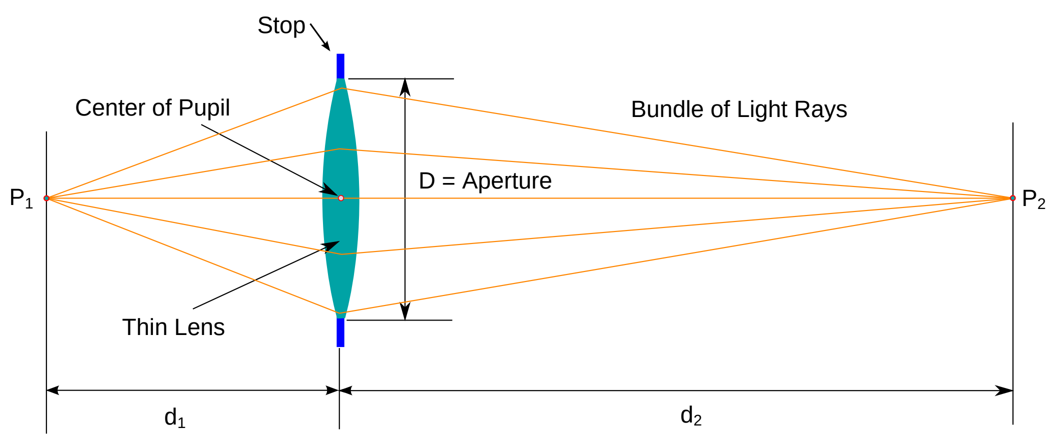

Figure 8. Thin Lens Focusing a Point

The focal length and image distances for a thin lens are related by the equation:

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f} \tag{2}$$

Solving for focal length gives:

$$f = \frac{1}{\frac{1}{d_1} + \frac{1}{d_2}} \tag{3}$$

Notice that as d_2 goes to ∞, d_1 approaches the focal length, f.

4 Imaging

Lenses have the remarkable ability to form images of objects or fields of view. There are three important characteristics of the imaging property of lenses:

1. It is reciprocal between sides of the lens. That is, in Figure 9, O_1 can be the object, and O_2 the image, or the other way around.
2. The image or object orientation is inverted by passing through the lens. Notice that the arrows shown in Figure 9 have different directions on either side of the lens. This is a direct consequence of the ray nature of light, as can be seen by tracing the rays from the tip and the tail of the arrows through the lens.
3. The image size relative to the object size varies as the distance of the object from the lens varies. Of course, the d_1, d_1' distance also changes to achieve focus. The focus equation, given in Equation 2, applies. See also Section 9.

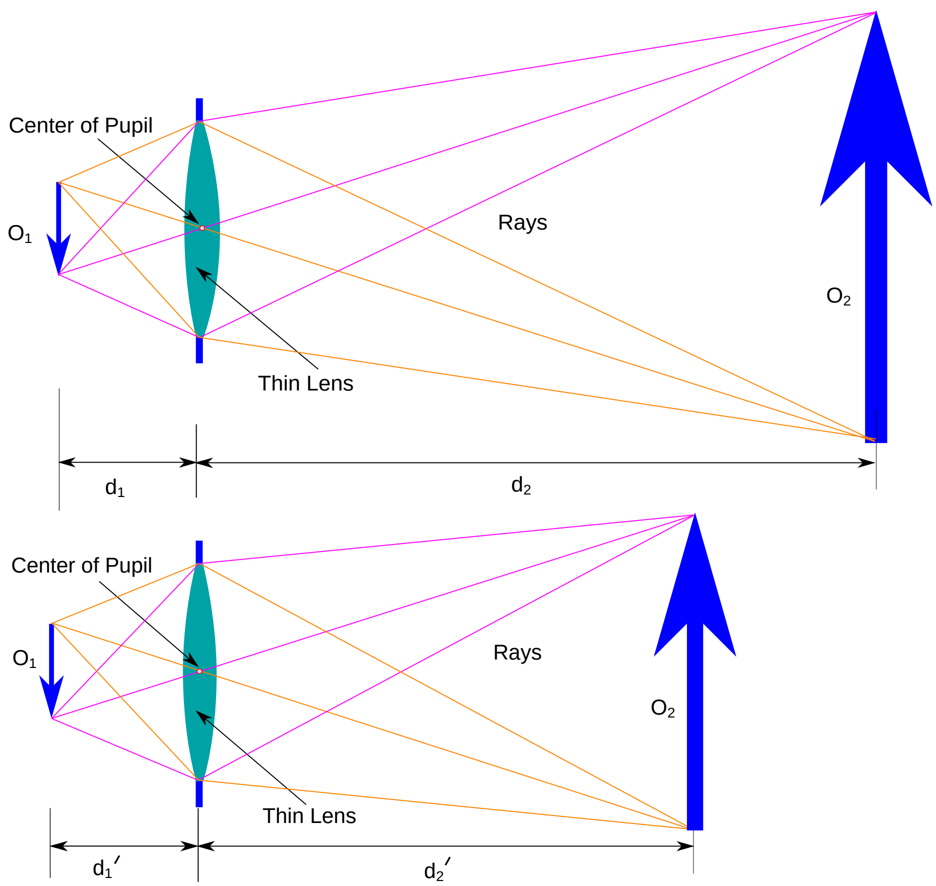

Figure 9. Thin Lens Forms Image of Object at Two Distances, d_2 and d_2'

It is common to see a projection system (projector, light engine) specified with a certain **throw ratio**. This is a simple concept, which describes the size of a projected image with respect to the distance from the projection lens.

5 Offset

Offset is a measure of the shift of the DMD with respect to the optical axis. Offset is introduced in a light engine to allow the projector unit to sit on a table and project its image entirely above the surface of the table. In a 0% offset design light engine, the center of the DMD is aligned exactly with the optical axis of the projection lens. This means that the image of the DMD is projected equally up and equally down from the optical axis (the direction the projector lens is pointing). This means that if the projector is sitting on a table, the bottom half of the projected image is blocked, and only the top half of the image reaches the screen.

By contrast, in a 100% offset light engine, the DMD is dropped until its upper edge (bottom edge of the image — remember, the image is projected upside-down) is aligned with the optical axis of the projection lens. This means that the entire image of the DMD is projected above the surface of the table, and the entire image can reach the screen.

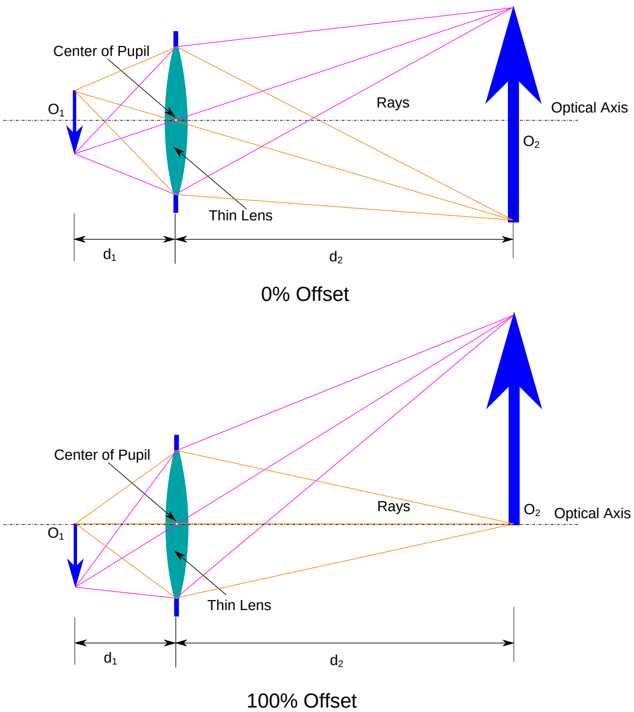

Figure 10. Offset

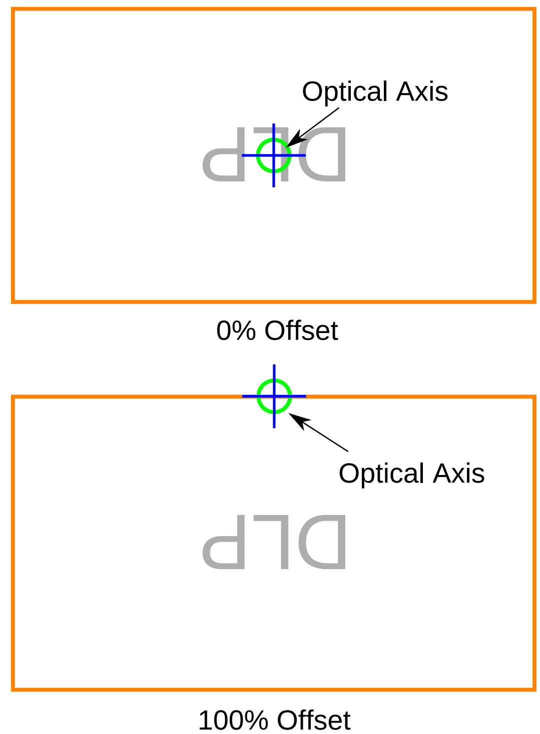

Figure 11. DMD Placement With Respect to Optical Axis

Both the DLP LightCrafter™ and DLP LightCrafter 4500™ light engines are 100% offset.

6 Aperture

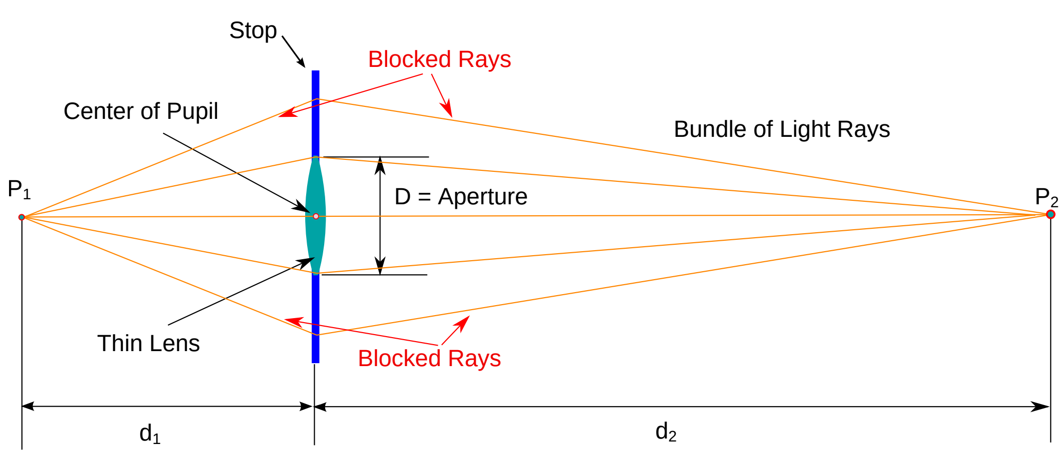

Figure 12. Smaller Aperture Lens Collects Less Light, Note Blocked Rays

The amount of light which passes through a lens depends on the area of the pupil, which is determined by the size of the aperture. The aperture may be determined by the size of the lens, or by a stop which limits the size of the aperture. The area of the pupil, A is given by the equation:

$$A = \pi r^2$$

where
- A is the area of the pupil
- r is the radius of the aperture (4)

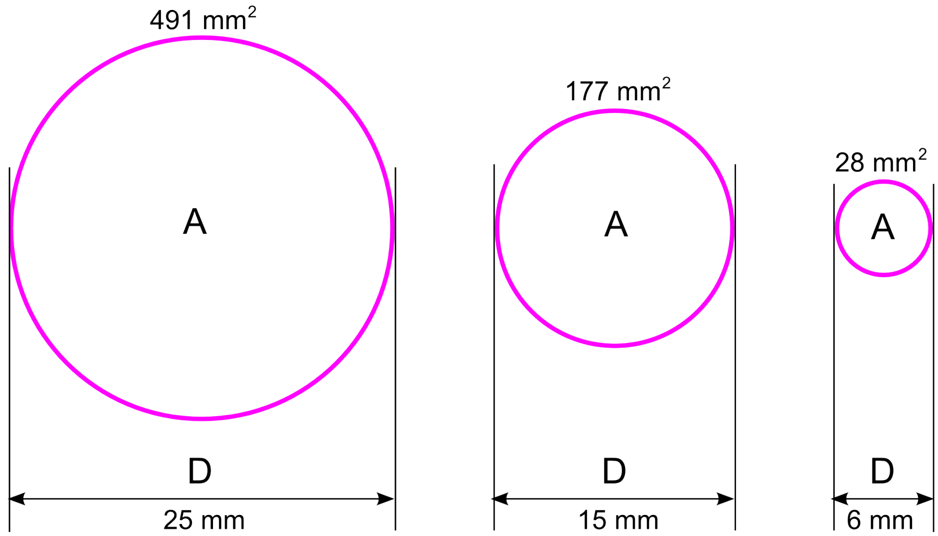

Figure 13. Apertures of Several Diameters With Areas

For example, if the aperture (diameter of the pupil) is 25 mm,

$$A = \pi\left(\frac{25}{2}\right)^2 = 156.25\pi = 491\,\text{mm}^2 \quad (5)$$

The size of the aperture (lens) chosen for a given application depends on the size of the DMD and the light output of the light engine. For example, the DLP LightCrafter light engine has a lens with about 6-mm aperture, corresponding to the smallest aperture in Figure 13. The DLP LightCrafter 4500 has a lens with an aperture of about 15 mm, corresponding to the middle image in Figure 13. The 25-mm aperture would be used in a larger light engine, probably with a larger diagonal DMD, such as the DLP5500 or DLP7000.

6.1 F-Number

It is evident that doubling the size of the aperture increases the area of the pupil by 4×. Conversely, reducing the aperture by half decreases the area of the pupil to 0.25×. A larger aperture gathers more light than a smaller aperture. However, the brightness of the image formed by a lens is not just a function of the area of the aperture. It also depends on the focal length of the lens. The size of the image formed by a lens is determined by its focal length. See Section 4 to learn more about imaging.

The relative brightness of an image formed by a lens is determined by the ratio of the focal length to the aperture of the lens. This value is often represented as the f-number of a lens.

$$\text{f-number} = \frac{f}{D} \quad (6)$$

For example, if a lens has an aperture of 20 mm, and a focal length of 40 mm, it has an f-number of 2.5. This is often written as f / 2.5. The relative brightness of lenses is inversely proportional to the square of the ratio of their f-numbers. For example:

Lens 1: f / 2.5

Lens 2: f / 1.8

$$I\left(\frac{\text{Lens1}}{\text{Lens2}}\right)=\left(\frac{1}{\left(\frac{2.5}{1.8}\right)}\right)^2=\left(\frac{1.8}{2.5}\right)^2=0.5 \tag{7}$$

This example shows that a lens f / 2.5 is one-half as bright as a lens of f / 1.8.

Strictly speaking, the f-number calculated above is valid only when either the image or the object is located at ∞. However, it is still helpful to consider at the distances involved in projection.

The DLP LightCrafter 4500 has a f-number of f / 2.1. The f-number for the DLP LightCrafter is not given in its documentation.

7 Focal Length

The focal length of a lens is the distance at which the lens focuses a point at an infinite distance (∞). This is considered a fundamental characteristic of a given lens. Note that the focal length is simply the limit for d_1 as $d_2 \to \infty$ using the equation given previously for focal length.

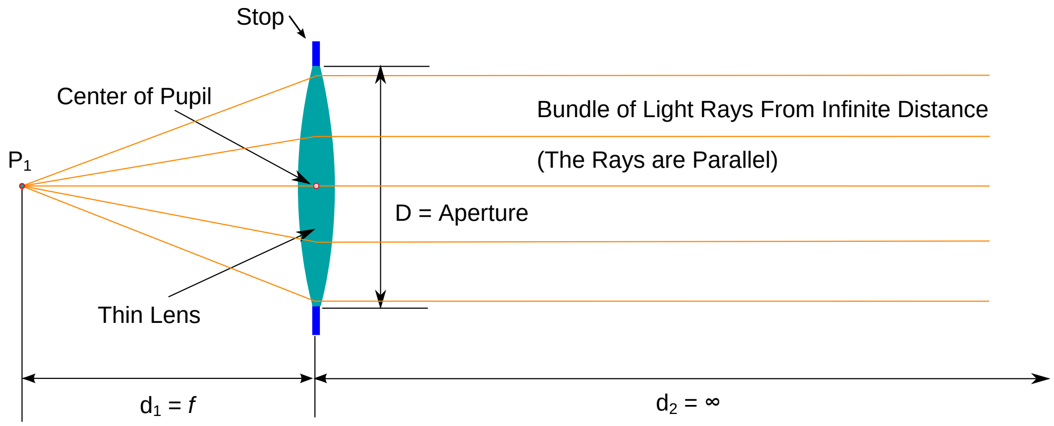

Figure 14. Image of a Point at Infinity Showing Lens Focal Length

The focal length also determines the size of the image formed by a lens.

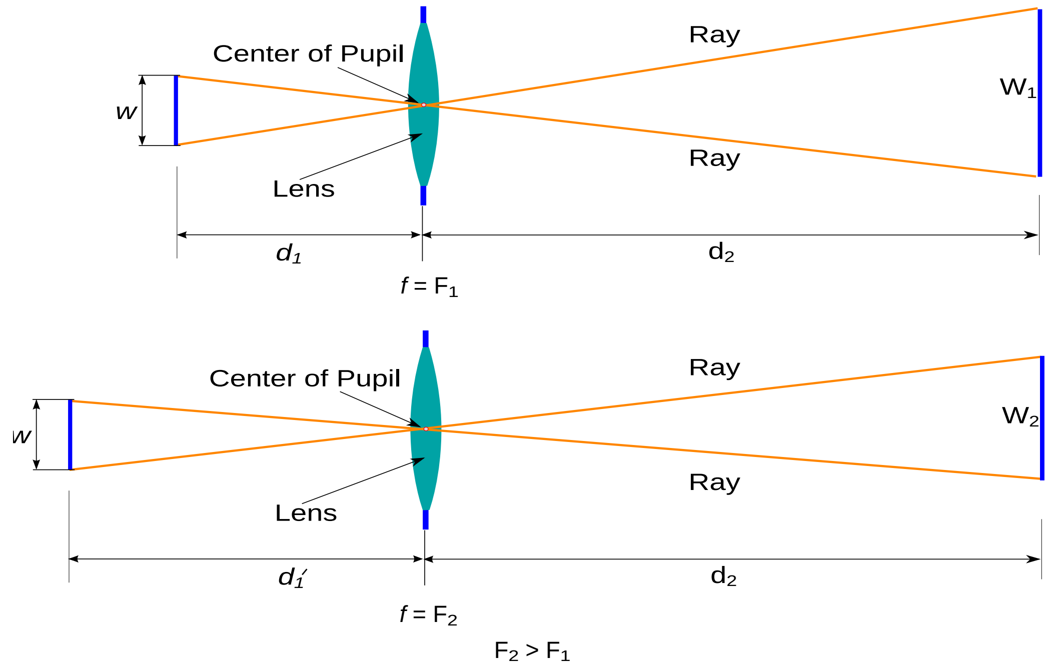

Figure 15. Image Size is Determined by the Focal Length of the Lens

Notice that $F_2 > F_1$, while the object (DMD) to the left of the lens remains the same size and the image at the same distance to the right of the lens (at the screen) is smaller. That is, the longer focal length lens has a smaller throw ratio with respect to the shorter focal length lens (see Section 8).

8 Throw Ratio

The throw ratio is the distance to the projection screen divided by the width of the image on the projection screen:

$$T = \frac{D}{W}$$

where

- **T** is the **Throw Ratio**
- **D** is the distance to the image on the screen
- **W** is the width of the image on the screen (8)

Example:

Distance to the screen = 1.5 m

Width of the image = 1 m

$$T = \frac{1.5\ \text{m}}{1\ \text{m}} = 1.5 \quad (9)$$

Observe that if the image is wider for a given projection distance, the throw ratio is a smaller number. If the projection distance is longer for a given width of image, the throw ratio is a larger number. Notice that the throw ratio is dimensionless.

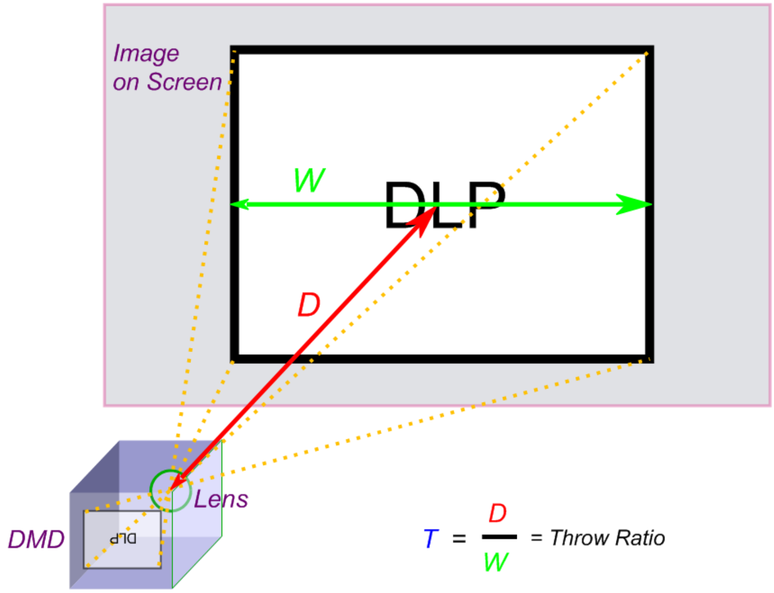

Figure 16. Throw Ratio

The DLP LightCrafter light engine has a throw ratio T = 1.66. The DLP LightCrafter 4500 light engine has a throw ratio T = 1.4.

8.1 Determining Focal Length from Throw Ratio

Many light engines reveal very few details of their optical design in their specifications. Usually all that is given is a throw ratio, and an f-number for the projection lens. Often, it is helpful to know the focal length of the projection lens. This value can be determined from the throw ratio.

For example, if the light engine specifications give a throw ratio of 1.8, this allows the geometry of the projection system to be sketched out. First, the width of the specific DMD must be known. As discussed previously in Figure 6, the width of the DMD active array is available in its data sheet on www.ti.com.

Note in Figure 17, that the triangles formed by the rays from the edges of the DMD to the edges of the image of the DMD on the screen are congruent. This means that:

$$T = \frac{d_1}{w} = \frac{D}{W} \tag{10}$$

For a given throw ratio, the distance from the DMD to the center of the pupil can be calculated by:

$$d_1 = T \times w \tag{11}$$

That is, the throw ratio times the width of the DMD gives d_1. Knowing d_1 and d_2 allows the focal length to be calculated by:

$$f = \frac{1}{\frac{1}{d_1} + \frac{1}{D}} \tag{12}$$

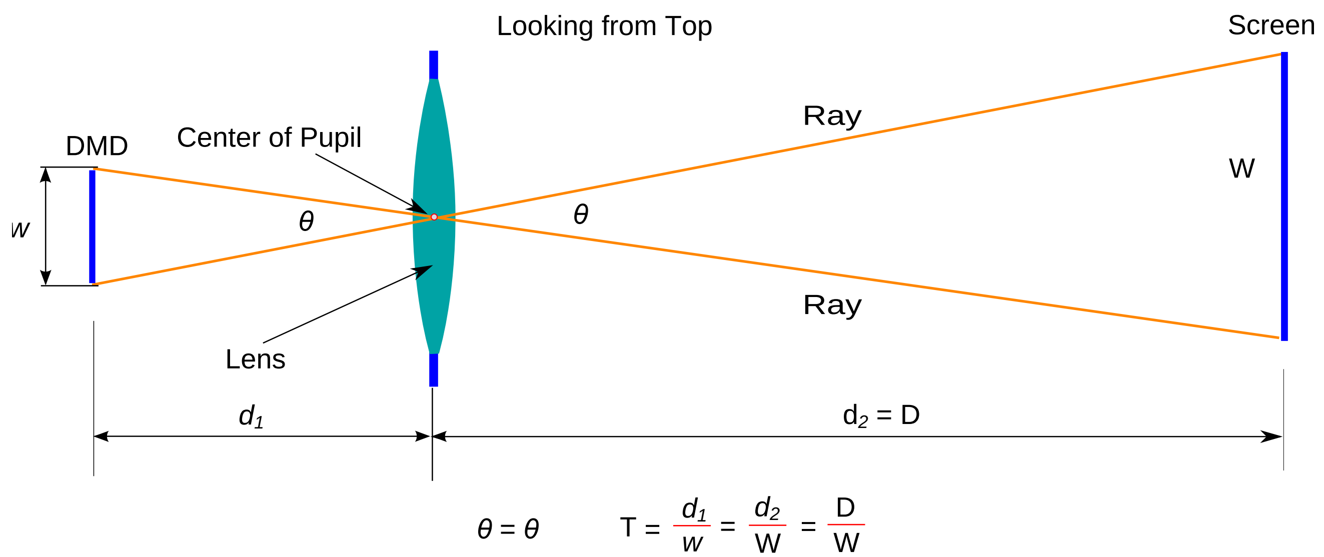

Figure 17. Relationship Between DMD and Throw Ratio

For example, suppose a light engine with a throw ratio of 2, and a DLP4500 DMD. The DLP4500 has a width of 9855 µm = 9.855 mm.

$$d_1 = 9.855 \times 2 = 19.7 \text{ mm} \quad (13)$$

Now, we must assume the distance at which the width of the DMD image on the screen was measured to determine the throw ratio. This is seldom given. So, we assume that the distance from the projection lens to the screen was 1000 mm.

$$f = \frac{1}{\frac{1}{19.71} + \frac{1}{1000}} = 19.32 \text{ mm} \quad (14)$$

This is an estimate, but it should be very close to the actual focal length of the projection lens.

9 Magnification

Magnification is a value which relates the size of the image on the screen to the size of the DMD. The larger the projected screen image, the larger the magnification; the DMD size stays the same. The throw ratio relates the image width to the projection distance. That means that the image size varies linearly with distance. If at 1 meter, the image is 66.7 cm wide (T = 1.5), at 2 meters, the image is 1.33 m wide. Note that the projected image will also be one-fourth the brightness per unit area that it was at 1 m distance. This happens because the area of the projected image is four times greater, while the illumination, the amount of light on the DMD, has remained constant.

The magnification is calculated by the equation:

$$M = -\frac{d_2}{d_1} = -\frac{W}{w} \quad (15)$$

The sign of the magnification is negative, because the image is inverted with respect to the object.

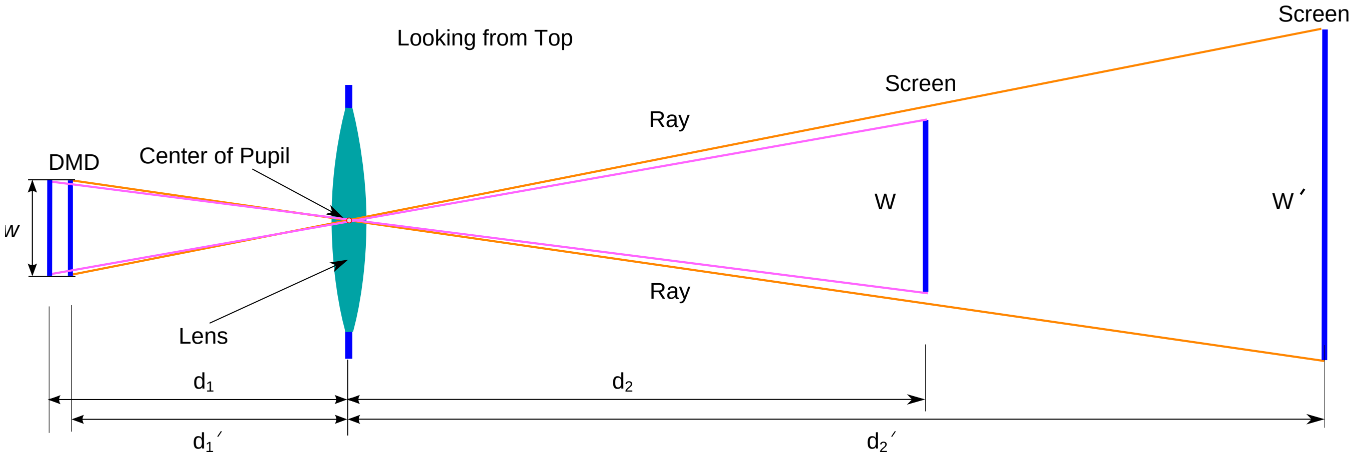

Figure 18. Projected Image Size Varies With Distance

For the following, we are only concerned with the absolute magnification, so we will neglect the image inversion (the negative sign). In Figure 18 the absolute magnification, in each case, is:

$$M_1 = \frac{W}{w} = \frac{d_2}{d_1} \tag{16}$$

$$M_1 = \frac{W'}{w} = \frac{d_2'}{d_1'} \tag{17}$$

For example, if the DMD is the DLP5500, and the width, W', is 1 m:

$$w = 11.059 \text{ mm} \tag{18}$$

$$W' = 1000 \text{ mm} \tag{19}$$

$$M_2 = \frac{1000 \text{ mm}}{11.059 \text{ mm}} = 90.4 \tag{20}$$

9.1 Pixel Size at the Screen

The apparent resolution of an image on the screen depends on the size of individual pixels at the screen image. This depends on the magnification (as described previously) and the actual pixel size on the DMD. Each pixel is a single micromirror on the DMD.

Consider the previous example with a magnification M_2 = 90.4. The DMD is a DLP5500, which has a micromirror size of 10.8 µm.

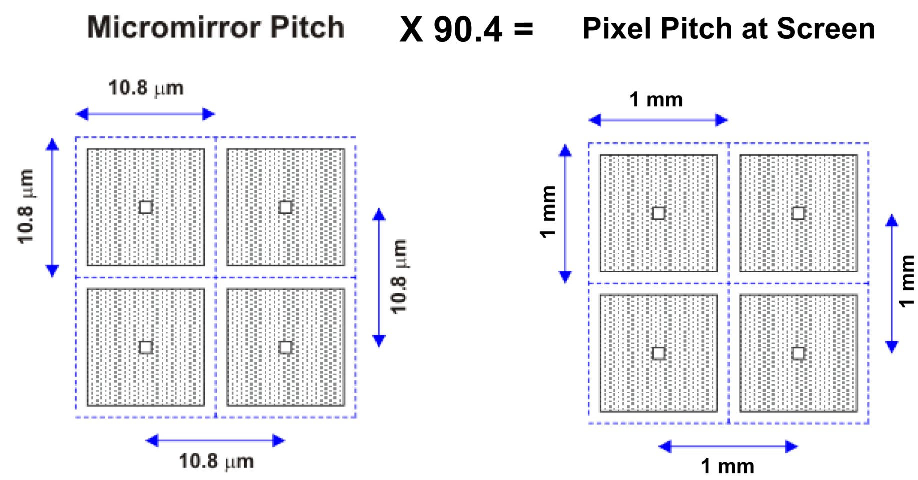

Figure 19. DLP5500 Micromirror Dimensions

The pixel size, P, at the screen, in this example, is:

$$P = 10.8\ \mu m \times 90.4 = 976\ \mu m\ =\ 0.976\ mm = 1\,mm \tag{21}$$

The pixel size is very nearly 1 mm. Given very good optics in the light engine, the micromirrors would be easily visible on a projection screen at close examination, but probably not from a normal viewing distance. For applications such as structured light, the calculation of pixel size at the working distance (equivalent to the screen distance above) is an important consideration in determining overall system measurement capability.

9.2 Demagnification

It is possible to project an image of the DMD which is smaller than the actual DMD. This is used in lithography and other applications when it is desired to reduce the size of the pixels and cover a very small image field. Examination of the lens equation (Equation 2) shows that if the distance to the DMD, d_1, is increased to exactly twice the focal length of the lens, the image formed at d_2 will be exactly the size of the DMD. That is, the magnification will be unity (1×), or actual size. If the DMD is moved to a greater distance, $d_1 >> f$, the image formed at d_2 will be smaller than the DMD. This means that demagnification is possible. In practice, the amount of demagnification is limited by the physical distance that the DMD can be separated from the lens, by numerical aperture (NA) considerations, and by the wave nature of light (diffraction).

10 Conclusion

Geometric optics can help with envisioning and preliminary design of DLP systems. Geometric optics are based solely on the procedure of ray tracing and give a good approximation of the characteristics of a DLP-based projection system. There is much more to optics: diffraction, aberrations, reflectivity, absorption, and wavelength (color), just to name some. However, familiarity with geometric optics can help a designer to “rough out” the overall dimensions of a DLP system. For these purposes, the contents of this application report should be helpful.

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