Technical Article

Quantifying Harmonic Distortion - Effect of sinc3 Filter Roll off (Part 3)

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In the second installment of this series, I briefly talked about the attenuation of frequencies due to the sinc[3] digital filter. In this installment, I'll quantify the theoretical degradation due to a sinc[3] filter and talk about how the Multiphase Power Quality Measurement with Isolated Shunt Sensors Reference Design reduces this degradation when calculating total harmonic distortion (THD).

First, let's begin by examining the equation of a typical sinc[3] filter show in Figure 1:

$$H(f) = \left(\frac{\text{sinc}\left(OSR\pi \frac{f}{f_M}\right)}{\text{sinc}\left(\pi \frac{f}{f_M}\right)} \right)^3 = \left(\frac{1}{OSR} \times \frac{\sin\left(OSR \times \pi \times \frac{f}{f_M}\right)}{\sin\left(\pi \times \frac{f}{f_M}\right)} \right)^3$$

Figure 1. The Equation of a Typical Sinc³ Filter

Here f_M is the delta-sigma's modulation clock and OSR is the selected oversampling ratio. The effective sample rate of the sigma-delta converter is equal to f_M/OSR.

From Figure 1, you can see that the frequency response depends on the sample rate of the sigma-delta analog-to-digital converter (ADC). Figure 2 shows the degradation of amplitude for a given frequency to ADC sample-rate ratio. The waveform in Figure 1 covers a range equal to half of the ADC sample rate, thereby covering the entire set of frequencies that the ADC could sense. In Figure 2, the waveform is normalized where the magnitude at each frequency is divided by the DC gain.

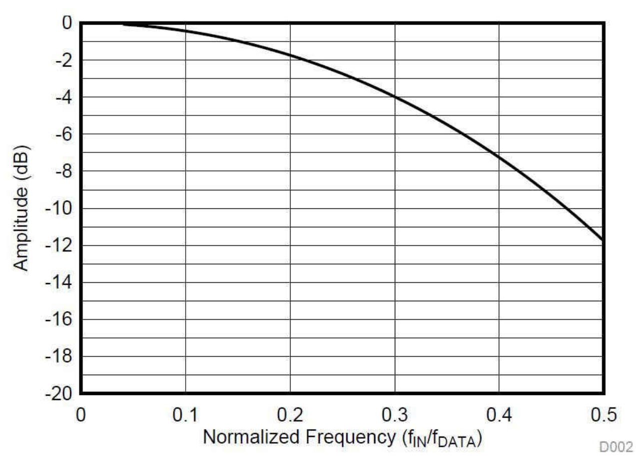

Figure 2. Sinc³ Amplitude Degradation over Frequency

In an energy-measurement system such as an e-meter, the system has calibration performed when applying a voltage and current waveform at the fundamental frequency. Therefore, any attenuation of the amplitude at the fundamental frequency will be accounted for during gain calibration of the system. Using this information, you can calculate the attenuation registered by the energy-measurement system at a specific frequency, f, by dividing the magnitude at that specific frequency, (H(f)), by the magnitude at the fundamental frequency, ($H(f_{fund})$).

As an example of the attenuation from the sinc^3 filter, let's say that you have the following test conditions:

- Fundamental frequency of 50Hz.
- Oversampling ratio of 256.
- An ADC sample rate of 4,096 samples per second (resulting in a modulation clock frequency of 1048576Hz for an OSR value of 256).
- Fundamental root-mean-square (RMS) voltage of 230V.
- Fundamental RMS current of 10A.
- Fifth-harmonic (frequency = 250Hz) RMS voltage of 23V.
- Fifth-harmonic (frequency = 250Hz) RMS current of 4A.

With these settings, the sensed contribution of the fifth harmonic scales by a factor of H(250)/H(50), which equals 0.982484702. Given these conditions, this means that the sensed fifth-harmonic current contribution is 3.929938809A, which has a percentage error of 1.782755276% from the actual contribution of 4A. As a result, the total sensed RMS current would be 10.74450646A instead of the actual value of 10.77032961A, which is a percentage error of -0.239761981%.

Similarly, the fifth-harmonic voltage contribution is 22.59714815V, which has a percentage error of 1.782755276% from the actual contribution of 23V. This means that the sensed total RMS value would be 231.1074017V instead of 231.1471393V, leading to a percentage error of -0.017191483%. Assuming the use of the THD_F definition of THD, the attenuation caused by the sinc^3 filter would cause a voltage THD reading of 9.824847021% instead of 10% and a current THD reading of 39.29938809% instead of 40%.

Applying the same test conditions but changing the fundamental frequency from 50Hz to 60Hz results in a fifth harmonic of 300Hz instead of 250Hz. The degradation in accuracy is worse than the 50Hz case. Specifically, these are the calculated parameter values:

- H(300)/H(60)= 0.974861507.
- Fifth-harmonic current contribution sensed: 3.899446027A (-2.513849325% error).
- Total RMS current sensed: 10.73339086A (-0.342967738% error).
- Current THD = 38.99446027%.
- Fifth-harmonic voltage contribution sensed: 22.42181466V (-2.513849325% error).
- Total RMS voltage sensed: 231.0903238V (-0.024579775% error).
- Voltage THD = 9.748615068%.

From these parameter values, you can see how there is a degradation in the results due to the attenuation of harmonics caused by the roll-off of amplitude of the sinc^3 filter at higher frequencies. This attenuation is worse when higher-frequency components are present, thereby resulting in less accurate results.

The Multiphase Power Quality Measurement with Isolated Shunt Sensors Reference Design demonstrates how to reduce the effects of roll-off of the sinc^3 filter. This reference design uses a successive approximation register (SAR) ADC for measuring phase voltages. For many applications, a SAR ADC can measure voltages without requiring multiple gain stages, since a large dynamic range of voltage is not often required. By using a SAR ADC instead of a delta-sigma ADC with a sinc^3 filter, there is no degradation of accuracy at higher frequencies caused by the converter.

For sensing current, the reference design uses the AMC1304M05 delta-sigma modulator with the MSP430F67641's sinc^3 digital filters. This modulator-plus-digital-filter combination can run with a modulation frequency as high as 20MHz. A high modulation frequency enables both a high OSR for accuracy and a high delta-sigma sample rate. The high delta-sigma sample rate has the benefit of reducing the magnitude of attenuation at higher frequencies as Figure 2 suggests, as it shows the degradation of amplitude for a given frequency to the ADC sample-rate ratio. In the reference design, the resulting delta-sigma ADC sample rate is 19,334 samples per second. Since not all of these samples are needed, one-fifth of the ADC samples for metrology calculations leads to a final sample rate of 3,866.8 samples per second.

Now let's see how the attenuation is affected by the higher sampling rate of 19,334 for the fundamental frequency of 60Hz. Assuming the same fundamental and harmonic current conditions as before, here are the calculated parameter values:

- H(300)/H(60) = 0.998859956.
- Fifth-harmonic current contribution sensed: 3.995439824A (0.114134518% error).
- Total RMS current sensed: 10.76863A (-0.015717239% error).
- Current THD = 39.95439824%.

Thus, you can see that increasing the sample rate reduces the magnitude of attenuation at higher frequencies, thereby leading to more accurate RMS and THD readings. In addition to reducing the effects of $sinc^3$ filter roll-off at higher frequencies, the reference design also uses shunts for each phase to reduce the degradation in accuracy present with current transformers at higher frequencies.

Additional Resources:

- Learn more on *total harmonic distortion* from our e2e blogs.
- Learn about the different options for calculating THD from part one of this series.
- Find the right reference design for your e-meter design by exploring our interactive diagram.